

# On the effects of sampling rate and interpolation in HMM-based dynamic signature verification

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## Abstract

*In this work, resampling techniques and interpolation are applied to on-line signatures. Their effect on the performance of an on-line signature system using Hidden Markov Models is studied. The presented techniques are based on linear interpolation and Catmull-Rom cubic splines. Experimental results are provided on the MCYT database comprising 16,500 signatures from 330 subjects and as many skilled forgeries. Our approach allows to reduce the sampling rate of the on-line signature capture system, leading to the reduction of storage resource requirements and increased simplicity without compromising and even enhancing the system performance. This also leads to the discussion of which is the minimum sampling rate for HMM-based dynamic signature verification.<sup>1</sup>*

## 1. Introduction

Biometrics has become a focus of interest both for research or commercial purposes in the last decades [1]. Among the great variety of biometric traits, signature is one of the most widely accepted, since it is commonly used for human-based authentication. Automatic signature verification is still a challenging task, and many approaches have been proposed in the last years [2, 3, 4]. This is primarily due to the great intra-class variability of signatures and the possibility of creating highly skilled forgeries. There are two main classes of signature verification systems: off-line and on-line systems. Off-line techniques use signature images as an input, considering only static information, while on-line signature verification systems use dynamic information of the signature, which has been captured and sampled

by a digitizing device [2].

Although some works have applied resampling techniques in the preprocessing stages of on-line signature verification systems [5, 6, 7, 8], signature resampling and interpolation has not been exhaustively studied in the literature. Moreover, while some authors report better results when using resampling techniques [7], some others have reported that resampling worsens the verification system performance as it eliminates relevant temporal data [5].

On the other hand, having on-line signature templates as compact as possible is of utmost importance in various applications, for example in resource-limited systems like smart-cards [9]. The interchange format of signature templates is currently under standardization [10]. Part 7 of the standard ISO/IEC 19795 defines the storage of time series information of the on-line signatures (e.g.,  $x$  and  $y$  spatial coordinates, time stamps, velocity, pressure, etc.). If interchange format based on time series is used (another possibility developed in Part 11 of the standard is the use of global features [11]), the storage size can be directly reduced by downsampling the time series. Although commercial systems provide very high sampling rates (100 samples per second and beyond), signature signals have maximum frequencies ranging from 20 to 30 Hz [2], so theoretically, sampling them at a higher rate than the Nyquist rate (40-60 Hz) would produce redundant information. The reduction of the sampling rate would alleviate the storage needs of the templates without theoretically affecting the verification performance.

In this contribution, we study the influence of different interpolation techniques for signature resampling in an on-line signature verification system based on HMM [11]. Our techniques allow to reduce the sampling rate of the signature templates by interpolating the missing samples, enhancing the verification performance in some cases. This is supported by experimental results on the large MCYT signature database [12], comprising 16,500 signatures from 330 subject, and as many skilled forgeries. The work is structured as follows. Interpolation techniques are ex-

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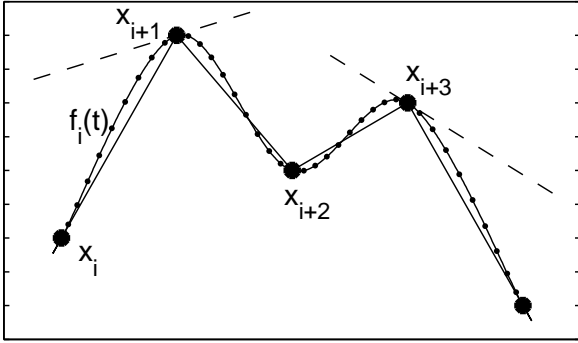


Figure 1. Catmull-Rom interpolation example

plained in Sect. 2. The database, the verification system used and the experiments conducted are described in Sect. 3. Results are finally discussed in Sect. 4.

## 2. Signature Resampling and Interpolation

In this work, two types of interpolation techniques have been implemented: Catmull-Rom cubic splines and linear interpolation. For each technique, two approaches have been considered. The first approach is an interpolation based on *equally spaced time intervals*, thus, introducing a fixed number of points between each pair of consecutive samples independently from their spacial distance. This approach preserves time dependencies, keeping a high spatial density of samples in slow strokes and a sparser sample distribution in high speed strokes. The second approach is designed to *reduce time dependencies* by inserting a number of points between each pair of consecutive samples directly proportional to their distance. Thus, no points are inserted between near samples and several points are inserted between separated samples.

### 2.1. Catmull-Rom splines

Splines are a set of functions commonly used for approximating or interpolating data points. A brief description of splines and their applications can be found in [13]. These functions are piecewise defined and different parameters can be chosen such as continuity, degree of approximation and curvature among others. In this work, Catmull-Rom cubic splines are used, as they interpolate the signature samples following smooth curve paths that are better adapted to the real signature strokes than linear interpolation. These type of splines were first described in [14] and have the following key properties:

- They interpolate the points from the data series (thus, the curves pass exactly through each point).

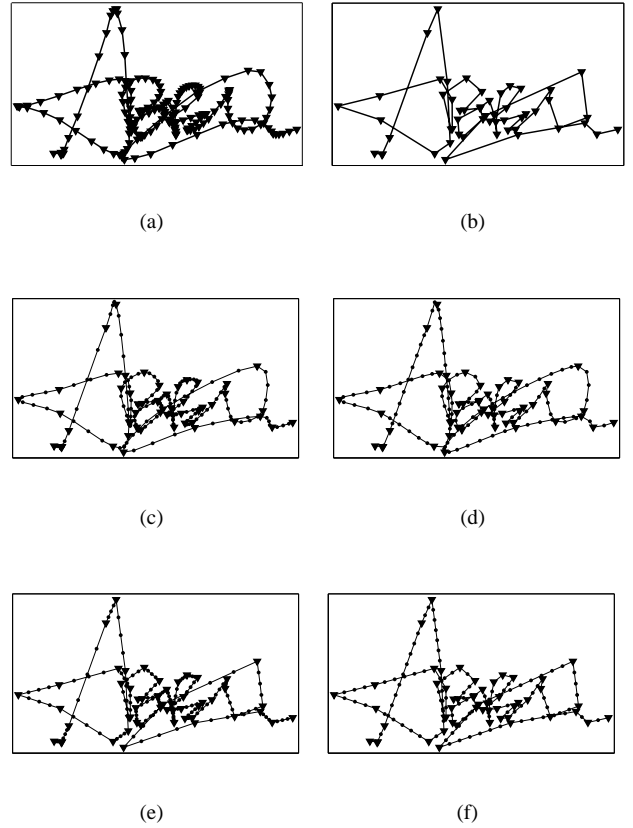


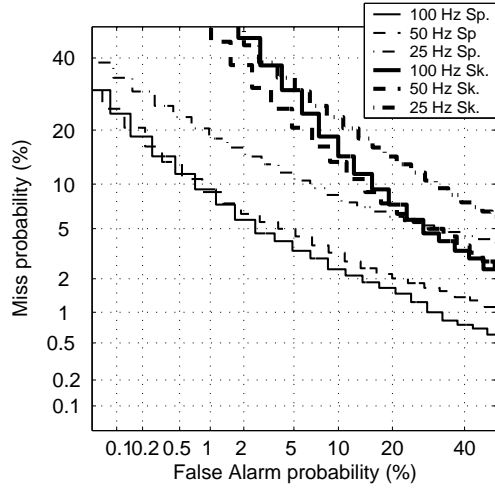
Figure 2. Example of interpolation of a signature downsampled to 25 Hz resampling it to 100 Hz. (a) Original signature, (b) Signature subsampled to 25 Hz, (c) Catmull-Rom interpolation, (d) Catmull-Rom interpolation reducing time dependencies ( $d = 100$ ), (e) Linear interpolation, (f) Linear interpolation reducing time dependencies ( $d = 100$ )

- The tangent vector at each point is parallel to the line connecting its two adjacent points.
- They have at least  $C^1$  continuity.

These properties can be observed in Fig. 1. Each segment or spline is defined by a parametric cubic curve in the form

$$f_i(t) = a_i t^3 + b_i t^2 + c_i t + d_i$$

In order to compute the parameters for the  $i$ -th curve segment, the following procedure is used [13]:



**Figure 3. Verification performance for different sampling rates. Sk = Skilled forgeries, Sp = Simple forgeries.**

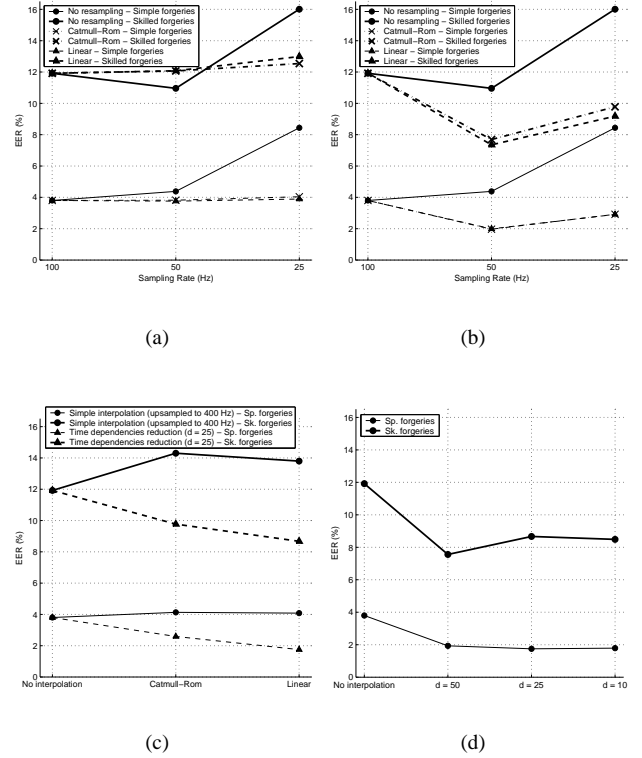
$$\begin{bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ x_{i+1} \\ x_{i+2} \\ x_{i+3} \end{bmatrix}$$

where  $x_i, x_{i+1}, x_{i+2}, x_{i+3}$  are four successive points (see Fig. 1). Note that Catmull-Rom splines provide equally time-spaced interpolating points which are not geometrically equally spaced.

## 2.2. Time dependencies reduction

The Catmull-Rom and linear interpolation schemes can be modified in order to reduce the time dependencies, leading to more homogeneous spacial spaced samples. This is achieved by varying the number of interpolated samples between each pair of samples proportionally to their Euclidean distance  $D$ . If the distance  $D$  between two consecutive samples is lower than a threshold of  $d$  points, no samples are interpolated between them. On the other hand, if the distance  $D$  between two samples is greater than  $d$ , the number of interpolated samples is equal to the number of times that  $d$  is contained in  $D$ .

In Fig. 4 the result of each interpolation technique is depicted for an example signature.



**Figure 4. System performance for each interpolation technique. (a) Normal interpolation, (b) Interpolation reducing time dependencies, (c) Interpolation without previously downsampling, (d) Linear interpolation reducing time dependencies without previously downsampling.**

## 3. Experiments

### 3.1. Database and experimental protocol

The signature corpus of the MCVT database is used in the experiments [12]. This database consists of 330 users, which are captured using a 100 Hz digitizing tablet. Signatures are captured in an enclosed area of  $17.5 \times 37.5$  mm, with a resolution of 100 points per millimeter. Thus, values span from 0 to 1, 750 points for the  $x$  coordinate and 0 to 3, 750 points for the  $y$  coordinate. For each user, there are 25 genuine signatures and 25 skilled forgeries. Skilled forgeries are produced by showing the target signatures to the forgers, letting them to train for several times and asking them to perform the signature with natural movements, avoiding slowdowns or interruptions.

We select the first 5 genuine signatures of each user as the training set, leading to  $20 \times 330$  genuine matching

Downsampled to	Sp. Forgeries EER	Sk. Forgeries EER
100 Hz (no downsampling)	3,80	11,92
50 Hz	4,38	10,96
25 Hz	8,44	16,01

(a) Original system performance (EER in %)

Interpolation Method	Downsampled to	Upsampled to	Sp. Forgeries EER	Sk. Forgeries EER
Catmull-Rom	50 Hz	100 Hz	3,82	12,07
Catmull-Rom	25 Hz	100 Hz	4,04	12,54
Linear	50 Hz	100 Hz	3,76	12,08
Linear	25 Hz	100 Hz	3,89	12,99
Catmull-Rom	-	400 Hz	4,13	14,30
Linear	-	400 Hz	4,08	13,80

(b) EER (in %) without reducing time dependencies.

Interpolation Method	Downsampled to	Parameter $d$	Sp. Forgeries EER	Sk. Forgeries EER
Catmull-Rom	50 Hz	100	1,97	7,69
Catmull-Rom	25 Hz	100	2,92	9,78
Linear	50 Hz	100	1,99	7,35
Linear	25 Hz	100	2,91	9,18
Linear	-	10	1,79	8,49
Linear	-	25	1,75	8,67
Linear	-	50	1,93	7,56

(c) EER (in %) reducing time dependencies.

**Table 1. System performance using different interpolation approaches**

scores. In order to test our system against random impostors, every signer is matched against one signature of each of the remaining users (i.e.,  $329 \times 330$  random impostor scores). Skilled impostors are tested using all the available forgeries, thus producing  $25 \times 330$  skilled impostor scores.

The experiments are structured in two groups: *down-sampling*, where the signature signals are downsampled to 50 Hz and 25 Hz, and then resampled to 100 Hz; and *up-sampling*, where the signals are only *upsampled* from 100 Hz to higher rates.

Down-upsampling experiments are performed using the following methods of interpolation: *i*) Catmull-Rom; *ii*) linear; *iii*) Catmull-Rom with reduced time dependencies ( $d = 100$  points); and *iv*) linear with reduced time dependencies ( $d = 100$  points).

In upsampling experiments, the original signature samples are interpolated using the following methods: *i*) up-sampling to 400 Hz with Catmull-Rom and linear interpolation, and reducing time dependencies in both methods with  $d = 25$  points; and *ii*) up-sampling with linear interpolation and reducing time dependencies for different values of parameter  $d$ .

For all experiments, the EER in % is used to compare the performance of different methods.

### 3.2. Signature Verification System

The verification system selected for the experiments uses only spatial coordinates ( $x$  and  $y$  coordinates versus time) and no pressure or pen inclination signals, which are available in the MCYT database. After some preprocessing (position and rotation alignment), and the computation of extended functions (path angle, velocity, curvature, acceleration, and time derivatives), similarities are computed using Hidden Markov Models. This system participated in the Signature Verification Competition 2004 with very good results [15] and is detailed elsewhere [11, 16]. The system performance for a sampling frequency of 100 Hz, and downsampled coordinate trajectories to 50 and 25 Hz is shown in Fig. 3. As can be seen, the system performance significantly decreases when the signature trajectory signals are downsampled to 25 Hz, while downsampling to 50 Hz leads to similar results than 100 Hz.

### 3.3. Experimental results

**Downsampling followed by Upsampling.** The results for up-sampling from 25 and 50 Hz to 100 Hz are shown in Fig. 4.a. As it can be seen, this approach leads to very sim-

ilar results than the ones obtained without downsampling. In Fig. 4.b, the results of interpolation with reduced time dependencies ( $d = 100$ ) are shown. The system performance increases significantly, specially in the case of re-sampling from 50 Hz, where the EER for simple forgeries is reduced more than a 50%, and the EER for skilled forgeries is also significantly reduced from its original value. For both cases (Figs. 4.a and .b), Catmull-Rom and linear interpolation perform similarly.

**Upsampling.** Fig. 4.c shows the results of using different methods of interpolation to upsample the signature signals to 400 Hz. Linear interpolation outperforms Catmull-Rom interpolation in all cases, reaching its best performance when time dependencies are reduced. Finally, the performance of linear interpolation with time dependencies reduction using different values for  $d$  is assessed, showing that the best results are achieved for  $d$  between 25 and 50 (see Fig. 4.d).

Experimental results are summarized in Table 1.

#### 4. Discussion and future work

The verification performance of a competitive HMM-based on-line signature system using different interpolation approaches has been compared. It has been shown that the performance can be significantly improved by reducing the sampling rate and interpolating the signal appropriately. Specifically, it has been shown that the captured signals, sampled at 100 Hz can be downsampled to 50 Hz (or directly captured at 50 Hz), without compromising the system performance and even increasing it. It has also been shown that using these techniques, signatures can be sampled at a lower rate than the Nyquist rate [2] while keeping the system performance under reasonable values.

Although Catmull-Rom interpolation provides visually similar signatures to the original ones, linear interpolation techniques have produced better results in most cases, while being simpler to compute.

Techniques that reduce time dependencies have provided the best results. Even when the signature is resampled above 100 Hz, time dependencies reduction techniques enhance the system's performance. This reduction of time dependencies may be also beneficial when combining the resampled HMM approach with other approaches based on global features related to timing information [11], as their complementarity may increase. This will be subject of further work.

Future work also includes performing an analysis of re-sampling and interpolation techniques in DTW-based on-line signature verification systems [7, 17], which are known to be heavily affected by these factors [8].

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