Realistic Synthetic Off-Line Signature Generation Based on Synthetic On-Line Data

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***Abstract*—A novel method for the generation of synthetic off-line signatures is presented. The proposed algorithm follows a two steps scheme: first, the raw synthetic dynamic functions of the synthetic signature are generated; second, several ink and paper models are applied to transform the on-line data to realistic static signatures. The novel approach is validated using four different publicly available databases both real and synthetic. The experimental protocol includes the comparison of both types of signatures in terms of: i) performance evaluation of two competitive and totally different verification systems; and ii) visual appearance according to human observers. The experimental results show the high similarity existing between synthetically generated and humanly produced samples, and the potential of the proposed method for the study of the signature trait.**

I. Introduction

One of the big challenges that the biometric-based security technology has to face nowadays is the permanent need for the collection of new data. These data should permit the objective and statistical evaluation of the performance of biometric recognition systems. In this context, one key element for the development of biometric applications is the availability of biometric databases.

However, the acquisition of biometric features corresponding to a large population of individuals, together with the desirable presence of biometric variability of each trait (i.e., multi-session, multiple acquisition sensors, different signal quality, etc.), makes database collection a time-consuming, expensive and complicated process, in which a high degree of cooperation of the donors is needed. Additionally, the legal issues regarding data protection are controversial [1], [2] and make the sharing and distribution of biometric data among different research groups or industries very tedious and difficult.

In this context, due to the difficulties linked to database acquisition and to the legal obstacles for their free distribution, in recent years different initiatives have been conducted within the biometric scientific community to generate databases formed by totally synthetic traits [3], [4], [5]. These synthetic databases present the advantages of: i) being effortless to produce (once the generation algorithm has been developed), ii) having no size restrictions (in terms of subjects and samples per subject) since they are automatically produced from a computer, iii) not being subdued to legal aspects because they do not comprise the data of any real user, and iv) eliminating human mistakes labelling the data which bias the performance evaluation of the algorithms. Nevertheless the final assessment have to be done with a real database.

In this work, we address the problem of generating synthetic databases of realistic human-like off-line handwritten signatures starting from synthetic dynamic data. Therefore, the generation process follows a two step protocol: i) first, fully synthetic on-line signatures are generated according to the approach introduced in [5] and validated in [6]; ii) then, the x and y dynamic information generated in the previous step is converted to off-line data using different ink and paper models in order to generate the final static database.

In order to validate the proposed approach for the generation of synthetic off-line signature, the problem to be faced is to determine a way to measure, in a quantitative manner, the *realism* of the synthetically produced samples. In the present work we have conducted two types of experiments:

- Experiment 1: Performance. The generated off-line signature databases should present the same inter- and intra-user variability as real signature datasets. This means that the performance of signature verification systems should be as similar as possible when it is tested on synthetic and real databases. Following this reasoning, we have compared the performance of two state-of-the-art off-line signature verification systems (working on totally different features and matchers), using two real databases and two synthetic datasets following the proposed scheme.
- Experiment 2: Appearance. The synthetic off-line signatures should look as close as possible to real signatures (i.e., they should have a signature-like visual appearance). Although this requirement is difficult to quantify as it partly depends on the subjective evaluation of the observer, we have carried out a human-aided perceptual experiment where real and synthetic signatures were rated by a number of volunteers.

The different results obtained show the high degree of similarity existing between the synthetic and real signatures and the suitability of the proposed technique for the automatic generation of fully synthetic off-line signature databases.

The rest of the article is structured as follows. The synthetic off-line signature generation method is described in Sect. II, with a brief subsection summarizing the generation of synthetic on-line data (Sect. II-A) and a more extensive one presenting the algorithm used to transform on-line samples in realistic off-line signatures (Sect. II-B). The experimental protocol is given in Sect. III, before presenting the results of the work in Sect. IV. Conclusions are finally drawn in Sect. V.

II. The Synthetic Off-Line Signature Generation Method

As already mentioned in the introduction, the synthetic off-line signature generation method is divided into two successive steps: in the first stage, the dynamic information is produced according to the methodology proposed in [5]; then in the last step, these dynamic data are converted into realistic off-line signature images following a procedure which includes different pen and paper models. Each of the two steps are described in the following sections.

A. Step 1: Synthetic Dynamic Signature Generation

Different dynamic signals such as the azimuth and elevation angles of the input pen might be considered as on-line information to model a signature. However, as the final goal in the present work is to generate synthetic off-line information, we will only characterize on-line signatures by three time sequences $[x[n], y[n], p[n]]$ specifying, respectively, the x and y coordinates, and the pressure p exerted during the signing process, at the time instants $n = 1, \cdots, N$.

The objective of this initial stage of the global generation algorithm is to produce the dynamic information (i.e., x, y and p functions) corresponding to different synthetic signers. In order to do this, a two-step methodology is followed. First, a signature-like graphic is generated following the spectral approach described in [7]. Although this first specimen has approximately the appearance and the pressure characteristics of a genuine signature, it does not possess many of the humanly produced kinematic characteristics of real writing. Thus, in order to confer this preliminary master signature with the velocity and acceleration properties of human strokes, it is processed in the second step of the algorithm using the Sigma-lognormal model [8]. The velocity function of the initial synthetic master signature is decomposed in singular strokes and the Sigma-lognormal parameters which best fit each of the individual strokes are computed. Then, the velocity function of the final synthetic on-line signature is reconstructed according to the previously computed parameters. The definitive coordinate signals x and y are finally obtained from the reconstructed velocity function.

These x and y signals, together with the pressure function p generated in stage one of the process, conform the final dynamic information that will serve as input for the second part of the synthetic off-line signature generation algorithm described in the next section (i.e., Sect. II-B).

For a detailed description and validation of the synthetic on-line signature generation process briefly summarized above we refer the reader to [5] and [6].

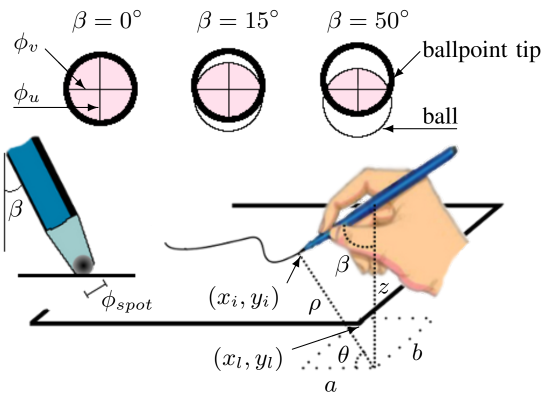

Fig. 1. Ink deposition model based on a human signing on a sheet with a ballpoint.

B. Step 2: From Dynamic to Static Signature

The objective of this second step of the generation method is to transform the synthetic dynamic information (i.e., x and y functions) produced in the previous stage, into realistic off-line signature images. The first task is to guarantee the signature continuity from the skeleton pixels. For this purpose, the signature coordinates x and y are 8-connected. Then the two main challenges to be faced are: i) model the type of pen and ink being used; and ii) model the type of paper on which the signature is being deposited.

1) Ink modeling: Conversion from on-line signature to off-line signature is carried out by using the ink deposition model introduced in [9]. That method models a ballpoint which is based on a ball inside a circle space (ballpoint tip). When the internal ball rolls along the stroke, the ink is deposited on the paper. The synthetic ink stroke is composed by superposition of individual ellipses. This approximation arises because pen does not touch perpendicularly the paper.

Individual ellipse appears on the intersection between the spherical ball and the ballpoint tip circle when the ballpoint is tilted to write, as shown in Fig. 1. The ellipse axis length (ϕ_u, ϕ_v) is directly proportional to the ballpoint tip circle diameter ϕ_{spot} and the ballpoint tilt. Concisely, the ellipse's minor diameter size could be approximated by $\phi_u = \phi_{spot} \cdot \cos\beta$, being β the ballpoint tilt, and the ellipses major diameter ϕ_v is, given ϕ_u, the distance between the intersection of both circles.

Angle β is calculated for a right handed writer as $\tan^{-1}(\rho/z)$, being z the height of the ballpoint tilt axis and ρ the Euclidean distance between the writing hand toehold approximate by $(x_l - a, y_l - b)$ and the written dot (x_i, y_i). It supposes that (x_l, y_l) is the lower right corner of the signature and the distance between the hand toehold and the lower right signature corner offset estimated as $a = b = 2.5\,cm$. Once worked out β, we are able to obtain the elliptical spot corresponding to the signature dot (x_i, y_i). As the ellipse is perpendicular to the ballpoint, it is rotate $\frac{\pi}{2} - \theta$, being $\theta = \tan^{-1}(x_i/y_i)$.

In our skeleton signature image (that produced in the dynamic generation step), each pixel of the signature trajectory is replaced by its elliptical spot adding the overlap of consecutive spots. The ink intensity inside the ellipse is modelled by

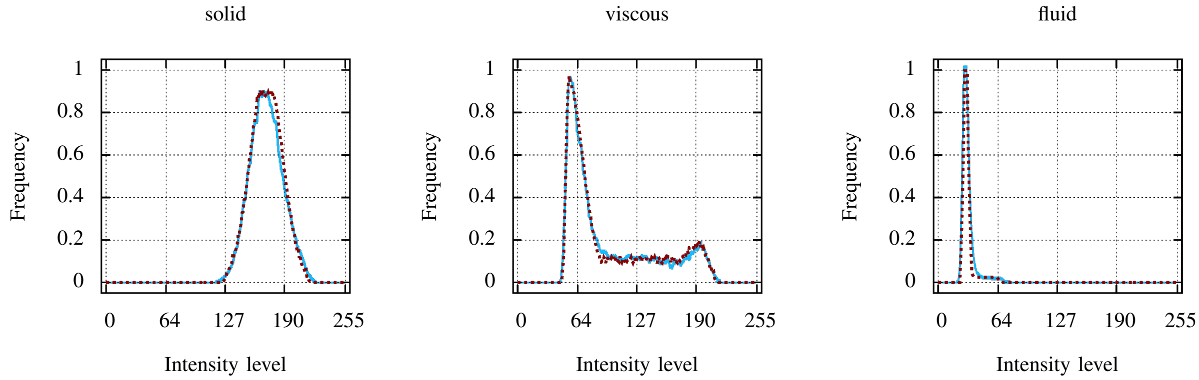

Fig. 2. Intensity Frequency plots for different types of ink. The solid line represents the model used in the present work, while the dashed line represents experimental curves obtained in [10].

a 2D Gaussian with an amplitude proportional to the pen pressure. As the resultant image presents no realistic dark areas where the stroke crosses due to the overlap add algorithm, the maximum dark value is set to $2\phi_v$, which saturates the ink level. Additionally, the image obtained is too regular in contrast to real writing where the ink deposited on the paper is usually noisy due to the irregularities of the ballpoint. Therefore, the spot is multiplied by a random spot to simulate the noise.

The last step to improve the realism of the off-line signature image consists in approximating the stroke grey level histogram to a real ink histogram distribution. Franke *et al.* [10] studied the ink histograms of the three more usual inks: solid, viscous and fluid. So, three curves approximating the histograms given by Franke *et al.* are generated (see Fig. 2). The histogram of the generated signature is modified to fix the ink randomly selected. A final smooth stage is carried out by using a 3×3 Gaussian mask.

In our model we have included the most usual commercial ball point diameters, that is $\phi_{spot} = [0.20, 0.25, 0.30, 0.35, 0.40, 0.45]$ mm. These six sizes, combined with the three different ink types simulated, results in a total of 18 available types of pens for the generation of synthetic off-line signatures.

2) Paper modelling: After the pen characterization, the second key aspect related to the transformation from the original synthetic dynamic information to the final realistic off-line signature image is the modelling of the paper. Often, the scanner devices are the key tool to acquire off-line handwritten signature. Since, in most cases, they introduce a non negligible noise level in the process, the synthetic signature should be slightly distorted as well. Therefore, a simple paper model was developed to improve the realism of the images. A uniform random image of values between 0.9 and 1 is generated and dilated with a disk structuring element of radius 10. It models the granularity paper to get the desired effect. The resulting image, which looks like a scanned sheet, is multiplied by the signature.

In Fig. 3 we show several examples of the initial synthetic on-line signature (generated in the first step of the process as described in Sect. II-A) and its corresponding off-line samples for three different types of inks (generated in the second step of the process as described in Sect. II-B).

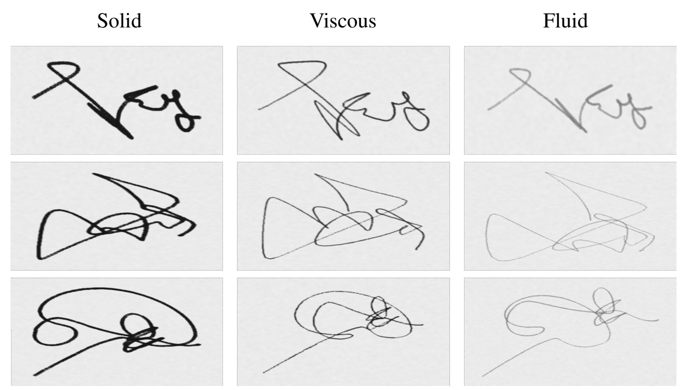

Fig. 3. Synthetic off-line signature examples generated with three different ballpoints and types of inks for three users

III. Experimental Protocol

The experimental framework has been designed to establish the level of compliance of the novel synthetic generation scheme proposed, with the twofold challenge posed by the problem of generating artificial traits: i) if automatic off-line signature recognition systems "see" the synthetic signatures as real; and ii) if humans perceive the synthetic samples as real.

To this end, the protocol comprises: on the one hand, tests aimed to evaluate the performance of signature verification systems both on real and synthetically generated databases (i.e., challenge i); and, on the other hand, experiments where the appearance of real and artificial samples are visually compared by human observers (i.e., challenge ii).

Therefore, in order to be able to carry out the sets of experiments presented above, the protocol includes both real and synthetic off-line signature databases and automatic recognition systems working on different sets of features and matchers. These databases and systems are presented in the following sections.

A. Databases

In order to reach meaningful conclusions regarding the validation of the proposed generation method, two publicly available real databases and two synthetic off-line signature databases are used:

- **Real DB1: MCYT-75 Signature DB** [11]. This dataset includes 75 signers collected at four different Spanish universities. The corpus includes 15 genuine signatures acquired in two sessions. All the signatures were acquired with the same inking pen and the same paper templates, over the WACOM Intuos A6 pen tablet. The paper templates were scanned at 600 dpi with 256 grey levels. The database is distributed by the Biometric Recognition Group-ATVS from UAM[1].
- **Real DB2: GPDS-960 Signature DB** [12]. This dataset contains 24 genuine signatures from 881 individuals acquired in one site in just one session. For the current work, only the first 350 users of the database were considered in the experiments. The repetitions of each genuine signature were collected allowing each donor to use his own pen on sheets of white A4 paper. Each sheet provided two different box sizes for the signature. The sheets were scanned at 600 dpi with 256 grey levels. The database is distributed by the Grupo Procesado Digital de Señales (GPDS) of the ULPGC[2].
- **Synthetic DB1: SSig-DB 1-Ink**. This dataset was produced following the proposed synthetic off-line signature generation method, and comprises 30 samples of 350 synthetic signers. All samples were generated with the $\phi_{spot} = 0.35\, mm.$ ballpoint and the viscous ink. The database may be obtained from the Biometric Recognition Group-ATVS website.
- **Synthetic DB2: SSig-DB Multiple Inks**. As the SSig-DB 1-Ink this dataset comprises 30 samples of 350 synthetic signers. However, in this case, samples were generated using the 6 standard ballpoint sizes given in Sect. II and three different types of inks. For each signature, both the ballpoint and the ink were randomly selected. The database may also be obtained from the Biometric Recognition Group-ATVS website.

B. Off-Line Signature Recognition Systems

In order to verify if the performance of automatic off-line signature recognition systems is similar when they are evaluated on real and synthetic data, in the experiments we have used two competitive systems based on totally different feature sets and matchers:

- **System A: Geometric features + HMM.** The signature is parametrized in Cartesian and polar coordinates. Both features are combined at score level. The Cartesian parameters consist of equidistant samples of the height and length of the signature envelope plus the number of times the vertical and horizontal line cut the signature stroke. In polar coordinates the parameters are equidistant samples of the envelope radius plus the stroke area in each sector. A multi observations discrete left to right HMM is chosen to model each signer features. The classification (evaluation), decoding, and training problems are solved with the Forward-Backward algorithm, the Viterbi algorithm, and the Baum-Welch algorithm. The initialization method is the equal-occupancy method. A detailed description of the system is given in [13].
- **System B: Local Binary Patterns + SVM.** In this case the Local Binary Pattern (LBP) operator has been used for static signature parametrization. Once the gray level image is transformed to code matrix which is divided into 4 equal vertical blocks and 3 equal horizontal blocks which overlapped by 60 %. From each block, we calculate the 255 bins histograms and the feature is obtained concatenating them. A Least Square Support Vector Machine (LS-SVM) with RBF kernel has been used as classifier. The system is described in [14].

[1]http://atvs.ii.uam.es/index

[2]http://www.gpds.ulpgc.es/download/

TABLE I. EER (IN %) FOR THE SYSTEM BASED ON GEOMETRIC FEATURES, EVALUATED ON DIFFERENT REAL AND SYNTHETIC DATABASES AND FOR DIFFERENT NUMBER OF TRAINING SAMPLES.

	Syst. A: Geom. + HMM (% EER)			
	#Signers	#Training Samples		
		2	5	10
MCYT	75	7.45	5.30	3.39
GPDS	75	6.65	3.88	2.59
SSig 1-Ink	75	8.19	4.50	3.08
GPDS	350	7.91	4.67	2.98
SSig 1-Ink	350	7.27	3.76	2.80

IV. RESULTS

As mentioned in previous sections of the present work, the experimental protocol has been designed with a two-fold objective: i) from a computer-based perspective, determine the performance of automatic recognition systems on real and synthetic off-line signature databases; ii) from a human point of view, establish the level of realism of the synthetic samples. In the next sections we describe the results reached in either of the experiments carried out to comply with these two objectives.

A. Experiment 1: Performance

Two main goals are pursued with this first experiment: i) on the one hand, determine if the performance of signature verification systems is similar when it is evaluated on real and synthetic databases; ii) on the other hand, estimate the influence of using multiple inks on the performance of off-line signature recognition systems.

To reach these objectives the two systems described in Sect. III-B are evaluated on the databases presented in Sect. III-A. Three different scenarios are considered depending on the number of randomly selected enrolment samples for each individual: 2, 5 or 10. In all cases genuine scores are computed matching all the remaining signatures of the same user against his trained model. Impostor scores are generated comparing all the samples from the other users against the trained model of the subject at hand. All the experiments are repeated 5 times changing the training samples in order to avoid biased results.

TABLE II. EER (IN %) FOR THE SYSTEM BASED ON LOCAL BINARY PATTERNS, EVALUATED ON DIFFERENT REAL AND SYNTHETIC DATABASES AND FOR DIFFERENT NUMBER OF TRAINING SAMPLES.

	Syst. B: LBP + SVM (% EER)			
	#Signers	#Training Samples		
		2	5	10
MCYT	75	2.28	0.35	0.26
GPDS	75	2.20	1.00	0.47
SSig 1-Ink	75	1.82	0.80	0.35
GPDS	350	3.14	1.46	0.76
SSig 1-Ink	350	2.13	0.71	0.29

The performance results (in terms of EER) of the two systems, evaluated on the two real databases and on SSig 1-Ink DB are shown in Tables I and II. For completion, Fig. 4 shows the DET curves of both systems for the three mentioned databases for the case of the evaluation carried out over 75 user with 5 training signatures per user. Different observations may be extracted from these results:

- From a quantitative point of view, the performance of the two systems is very similar when they are evaluated on real data (MCYT and GPDS) and on synthetic data (SSig 1-Ink). This is specially relevant since the two verification schemes work on conceptually totally different features and classifiers. This result points out the potential of the proposed synthetic generation method to be used as an initial tool to estimate the performance of automatic off-line signature recognition systems, avoiding this way the burdens associated to real databases (i.e., time consuming acquisition campaigns and legal protection data issues).
- From a qualitative perspective, the general behaviour of both systems is also fully comparable between real and synthetic datasets. For instance, in Tables I and II it may be observed that, similarly to the case of using genuine signatures, for the SSig 1-Ink DB the higher the number of training signatures the lower the EER of the system. This similarity in the performance of the systems is not restricted to the EER, but may generalized to the whole range of scores as shown in Fig. 4.
- In spite of the very significant similarities highlighted above, one difference should also be noted between the performance of both systems when it is evaluated on real and synthetic data. On real data, the EER tends to increase when the number of users of the dataset increases (e.g., between GPDS 75 users and GPDS 350 users). On the contrary, on synthetic data, the performance of the systems tends to improve (i.e., the EER decreases) when the number of synthetic users increases (e.g., between SSig 1-Ink 75 users and SSig 1-Ink 350 users). This observations suggests that the inter-user variability found in synthetic data is somewhat smaller than that present in real signatures.

Regarding the second objective of the present experiment, that is, study the influence of the ink in the systems performance, results are given in Table III. Here, the performance of the two systems is compared, in terms of the EER, on the two synthetically generated databases: with just one ink (SSig 1-Ink), and with multiple inks (SSig M-Inks). It may be observed that, while the performance of the geometry-based systems is barely affected by the use of multiple inks (e.g., EER raises from 3.76% to 4.28% for the case of 5 training signatures), in the case of the LBP-based system the performance drastically drops (e.g., EER increases from 0.71% to 3.64% for the case of 5 training signatures). These results are consistent with what was expected prior to the experiment since the change of ink does not have a big effect on the general geometry of the signature (System A), but has a very deep impact on its grey levels (System B).

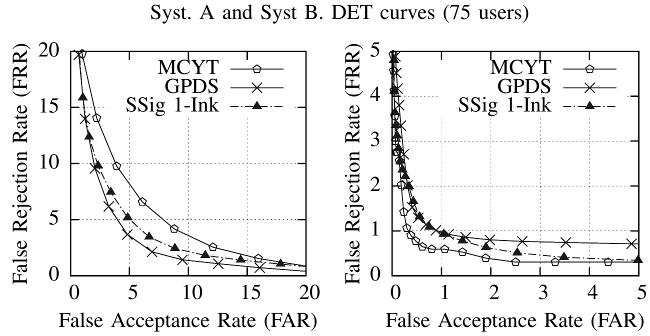

Fig. 4. DET curves corresponding to the geometry-based system (left) and the LBP-based system (right), evaluated on the MCYT, GPDS and SSig 1-Ink databases. These curves correspond to the evaluation carried out over 75 users with 5 training signatures per user.

TABLE III. EER (IN %) FOR THE TWO SYSTEMS CONSIDERED IN THE EXPERIMENTS, EVALUATED ON THE SYNTHETIC DATABASES GENERATED WITH ONE INDIVIDUAL INK AND MULTIPLE INKS.

	System	#Training Signers	#Training Samples %		
			2	5	10
SSig 1-Ink	Geom.+HMM	350	7.27	3.76	2.80
SSig M-Inks	Geom.+HMM	350	7.76	4.28	3.68
SSig 1-Ink	LBP+SVM	350	2.13	0.71	0.29
SSig M-Inks	LBP+SVM	350	5.72	3.64	2.78

The previous observations suggest that, although the LBP-based system is really competitive, some ink normalization scheme should be developed to reduce the impact of ink-variability on its performance. Such a normalization approach could be based on the robustness of geometry-based features to ink changes.

B. Experiment 2: Appearance

This second experiment is designed to evaluate from a statistical point of view the subjective perception that human observers have of synthetic off-line signatures generated following the method proposed in the present work. For this purpose, a set of 50 real and 50 synthetic samples was given to a group of 35 people with some expertise on signature recognition (most of them work in research laboratories related to biometrics). The participants were asked to mark each specimen from 0 (fully synthetic) to 10 (fully real) according to their impression after a quick inspection of the signature. The maximum time permitted to complete the experiment was 20 minutes.

Two types of errors can be committed in the classification task: *i*) a real signature is marked as synthetic, measured by

TABLE IV. Error rates, average score and average time of the 35 participants in the appearance evaluation experiment. FSR stands for False Synthetic Rate, FRR for False Real Rate, and ACE for Average Classification Error.

Proficient Participants (35)					
Error Rates (%)			Average Score		Average Time (minutes)
FSR	FRR	ACE	Real	Synthetic	
22.90	21.40	22.15	7.38	2.79	9.34

the False Synthetic Rate (FSR), and ii) a synthetic signature is mistaken with a real sample, measured by the False Real Rate (FRR). The final Average Classification Error (ACE) is defined as $\text{ACE} = (\text{FSR} + \text{FRR})/2$. These error rates are presented in the first three columns of Table IV. In the next two columns we give the average scoring given by all subjects to the 50 real and synthetic samples. Finally the average time taken to complete the experiment is shown.

We can observe that over 22 % of the signatures were misclassified, proving the real-like appearance of synthetic samples (a random classifier would present an ACE of 50 %). It should also be noticed that both error rates FSR and FRR are very close (22.90 % and 21.40 %, respectively) which means that the number of mistaken real and synthetic samples is very similar and that it is not easier to distinguish one class over the other. Furthermore, the average score given by the participants to real and synthetic specimens is not too far apart, reinforcing the idea that human subjects have a very similar perception of both types of signatures.

V. Conclusions

In this work a novel method to generate realistic synthetic off-line signature databases starting from artificial dynamic data has been presented. The method has been validated on two real and two synthetic databases: one generated modelling only a single ink and the other one produced simulating multiple inks. The validation protocol included two types of experiments where synthetic and real signatures were compared in terms of: i) performance evaluation of two totally different off-line signature verification systems; and ii) visual appearance according to a punctuation given by human observers. In all the tests, the synthetic signatures obtained remarkable results, showing a very high degree of similarity with humanly produced samples in all the considered scenarios.

The validation protocol and results described in the present work have demonstrated that, from a computer-based recognition point of view, the databases produced following the proposed generation approach are fully representative of the different real signatures that may be found in every day life in a western-European context. From a human perspective, it is clear that some of the signatures have a more realistic appearance than others, however, the overall realism of the artificial signatures seems to be quite convincing.

The results described in this work have shown that the novel synthetic off-line signature generation method proposed constitutes a very powerful and useful tool with a great potential for many different tasks such as: performance estimation, security evaluation in order to test existing biometric solutions against fraudulent access attempts, individuality studies, or for synthetically improving the performance of recognition systems by generating further enrolment data or by complementing available on-line samples with off-line synthetically generated ones.

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